Problems

Ted Eisenberg, Section Editor

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before June 15, 2015

• 5349: Proposed by Kenneth Korbin, New York, NY

Given angle A with $\sin A = \frac{5}{13}$. A circle with radius 1 and a circle with radius x are each tangent to both sides of the angle. The circles are also tangent to each other. Find x.

• 5350: Proposed by Kenneth Korbin, New York, NY

The four roots of the equation

$$x^4 - 96x^3 + 206x^2 - 96x + 1 = 0$$

can be written in the form

$$x_{1,2} = \left(\frac{\sqrt{a} + \sqrt{b} + \sqrt{c}}{\sqrt{a} - \sqrt{b} + \sqrt{c}}\right)^{\pm 1}$$

$$x_{3,4} = \left(\frac{\sqrt{a} + \sqrt{b - \sqrt{c}}}{\sqrt{a} - \sqrt{b - \sqrt{c}}}\right)^{\pm 1}$$

where a, b, and c are positive integers.

Find a, b, and c if (a, b, c) = 1.

• 5351: Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania

Let x, y, z be positive real numbers. Show that

$$\frac{xy}{x^3 + y^3 + xyz} + \frac{yz}{y^3 + z^3 + xyz} + \frac{zx}{z^3 + x^3 + xyz} \le \frac{3}{x + y + z}.$$

• 5352: Proposed by Arkady Alt, San Jose, CA

Evaluate
$$\sum_{k=0}^{n} x^k - (x-1) \sum_{k=0}^{n-1} (k+1) x^{n-1-k}$$
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